A Prime Number Sieve
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Is it possible to construct a sieve that finds all prime numbers. It is within a limited range of integers, by using the Sieve of Eratosthenes(276-l96 BC). Essentially the method consists of listing all the integers up to a given limit N, and crossing off multiples of 2, 3, 5, 7, and so on. The first integer, 1, is ignored and 2 is examined. Every other integer is crossed off, thereby eliminating all integers divisible by The smallest remaining integer is 3. and ever y third integer is crossed off. This eliminates all integers divisible by 3. The process is continued with every smallest remaining integer (except those already considered, n, and crossing off every nth integer. The remaining integers will be all the primes up to the limit N. There are other sieves but the one to be shown here has several intriguing possibilities connected with it.

Since a prime number contains no integral multiple of other numbers. other than I and itself, one needs only to line up all the multiples of numbers. one above the other. to construct a complete sieve. The simplest and quickest way to do this is simply to draw lines with slopes 1/1, 1/2 1/3 1/4 … This has been done in Figure 1. Distances that are multiples of one unit can be found on the line corresponding to y = 1 where vertical, horizontal, and sloped lines intersect simultaneously. Similarly, distances that are multiples of two units can be found on the line corresponding to y = 2 at places where vertical, horizontal, and sloped lines intersect simultaneously.

To find if a number is prime first find the number on the line with slope l/1 in in Figure 1. Now go down the vertical line passing through this number and check for any simultaneous triple intersections. Any such intersection means that the number is not prime. but is some multiple of a smaller number. As an example, 19 is followed down from its position on the 1/1 line, There is no triple intersection. hence 19 is prime. If 18 is followed from its position. there are triple intersections at 9, 6 3, and 2, and meaning that it iS is a multiple of 9, 6, 3, 2 and certainly not prime. Actually, one needs to consider only as many points of triple intersection equal to or less than square root Of the number being examined.

After constructing this sieve it was found that wherever a triple intersection occurs a point is located that is a solution to some form of the general equation

Ap^m + Bp^n + Cp^o + … + Qp^z = Jq

where *p* and *q* are variable integers. *A. B, C, ...* are constant integer coefficients. and *m, n, o, ...* are integer exponents.

Glancing at this sieve it is easy to see how it can be used to multiply and divide numbers, to find all the factors of a number, to find the solutions of congruence's, and to find the various powers and roots of numbers. It is also possible to add and subtract numbers by drawing all the rest of the lines of slope 1/1. That way a number located between two numbers along *x* can be found by following along verticals and diagonals until its exact size is found on the *x* or y axes. Hence this graph can become an integer slide rule without moving parts. *The graph also shows slopes with numerator greater than one illustrated as dotted lines. Slopes with numerator greater than denominator show Pi, e, and infinity to illustrate that the tangent function is used for these slopes = side opposite/side adjacent = y/x.*

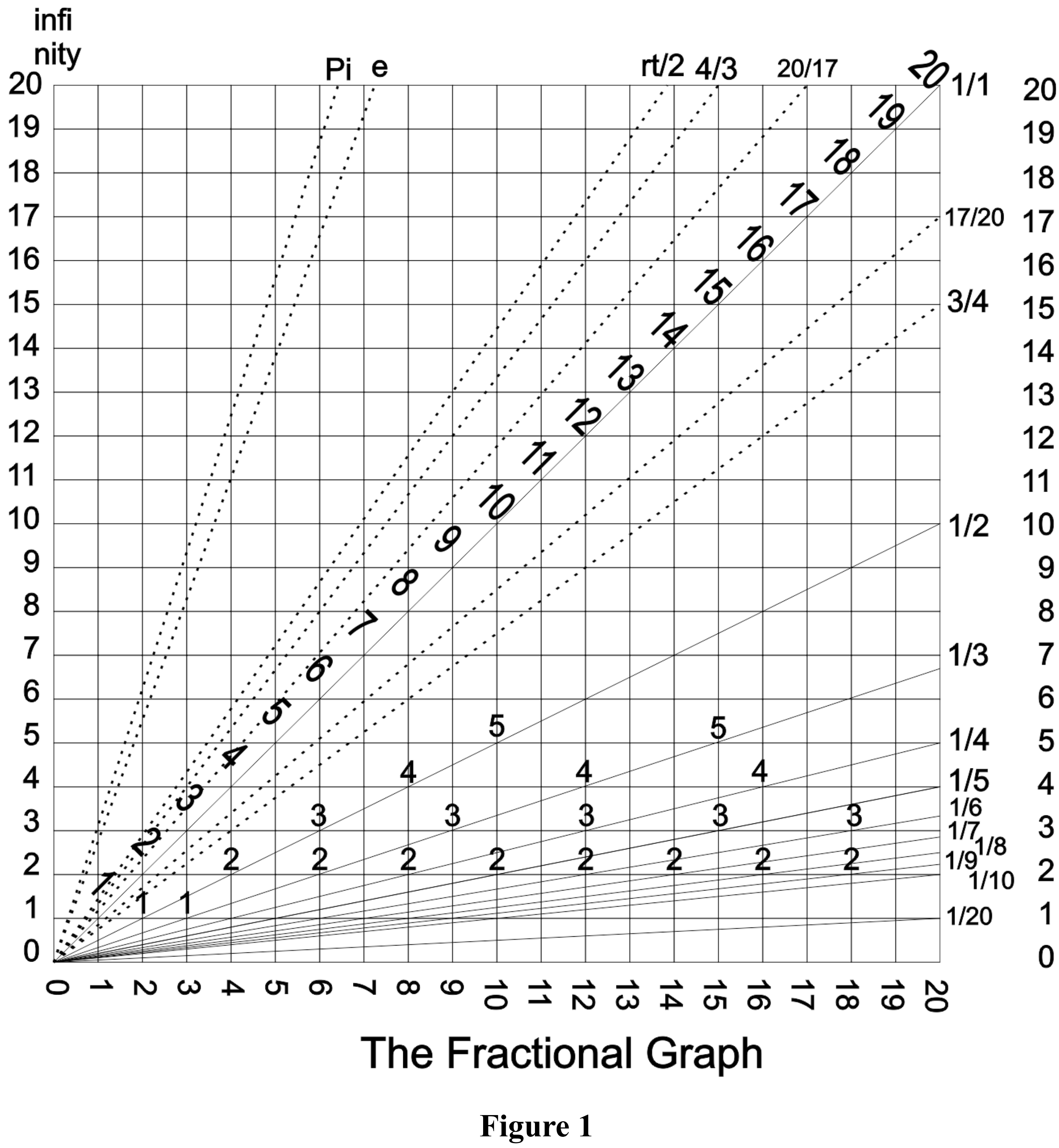

Figure 1

The sieve is very handy tor discovering the properties of integer equations. In the equation $x = y^2$, you can find the squares of **y b**y finding the slope with a denominator the same as y and finding the number *y* on this slope. The square of y will be found on the intersection of the vertical line passing through *y either* with the line of slope 1/1, *or with the x axis*. For example if y = 4, the square of *y* is found by going up the vertical line through number 4 on the line of slope 1/4 to the line of slope 1/1, where 16 is found.

You can find all sorts of interesting properties of integral equations, perhaps discovering *something very curious or puzzling*. One conclusion to be drawn from this graph is that points tend to cluster near the axes If *you think* of integral equations as able to occur at random the graph shows how more likely their points would occur near one or the other axis, rather than some place in between them.

It might be possible to set up laser beams (with a combination of lenses and mirrors only a few laser beams would he needed) traversing the axial lines and slopes so that interference patterns produced or bright spots would show relevant intersections of three beams. The August 7. 1967 issue of 'Electronics' pages 44-46 discusses how Stephen Harris, a Stanford electronics engineer created a tunable laser. The effect uses the principle of parametric fluorescence which was predicted several years ago. *Since very short wave lengths are possible and lasers are beams of coherent waves of light, which remain point like, computation with integers might be possible such as finding* ***primes*** *up to a given number. The basic problem would be miniaturizing the device so that fairly large primes could be produced. Where three beams intersect the light should be brighter indicating the intersection.*

Douglas A. Engel, 25, has B.S. in mathematics from Fort Hays University, Hays, Kansas. This article was written while working with an IBM 360 at Climax Molybdenum Co, the largest underground mine in North America, altitude 11300 feet. The original JRM article has been edited in a few places. I completely redrew the graph and shortened the wording about the laser computation idea. I am very interested by the fact that my Atomihedron puzzle http://www.puzzleatomic.com/ATOMIC%20pg1.htm produces rational fractional patterns very similar to this graph in a very symmetrical manner using precisely organized knots.