Cubo Game puzzle Reprinted with Permission of **Journal of Recreational Mathematics,** JRM, and Baywood Periodicals V 5 No. 3, 1972 pp-211-215. *(Cubo exhibits behavior similar to a twisty puzzle. One person was quite irate that there was no rule prohibiting players from reversing a previous move so that rule should be added to the game rules for Cubo.)*

Cubo-Caibo

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Cubo-Caibo is a matrix of cubical blocks. It is quite fascinating and is a good teaching device (or interest generating device) because of the recent emphasis on matrix mathematics.

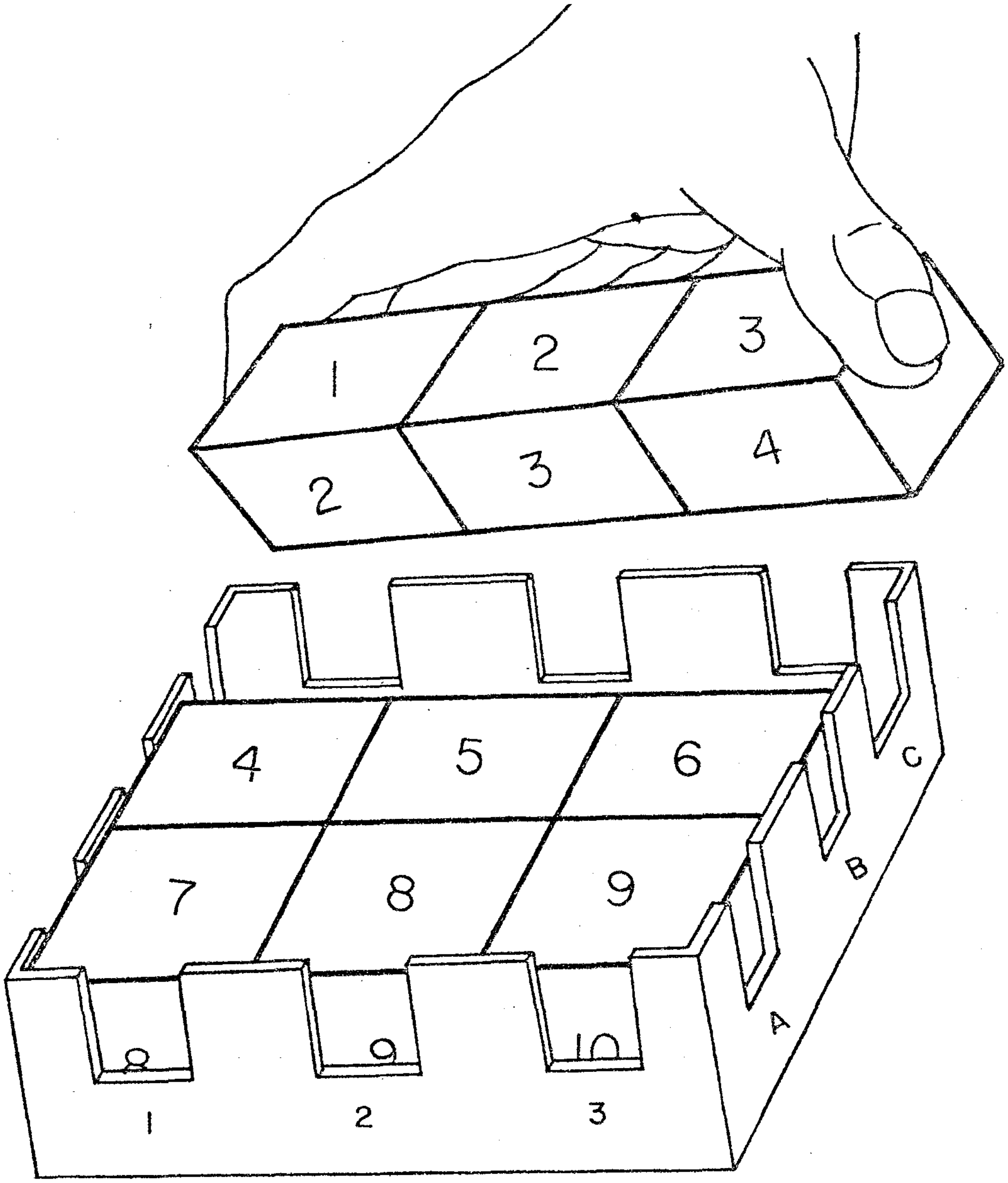
A cubical matrix is a square or rectangular array of cubes one layer thick. The cubes have numbers or colors on their faces. Each cube in the matrix is marked or colored identically. To work the matrix pick up a row or column of the cubes at a time and rotate them. Then replace them in the matrix. This can be done very easily by pressing the cubes together as you pick them up. Figure 1 should show the reader how to do it more fully. In a 2×2 or 3×3 matrix a row or column may be picked up, rotated and replaced with only a thumb and forefinger of one hand, if the cubes are not too large.

If the array is composed of cubes each colored identically with six different colors on their six faces then any combination of the colors can be achieved by rotating rows and columns 90° at a time.

If rotations are restricted to 180° only two colors can appear uppermost if the arrangement begins with only two colors uppermost. An additional property of the 180° restriction is that it perfectly preserves symmetry. This is illustrated by a set of 180° rotations in a 4×4 array in Figure 2.

A further property is the preservation of asymmetry with the 180° restriction. For instance in a 2×2 array if we begin with 3 blacks and a white face uppermost (assuming white and black colors are on opposite faces on individual cubes) it is impossible to get all black or all white faces uppermost. There will always be three of one color and one of the other. In larger arrays 1/4 of the cubes can be set so that at least 1/4 of them are always black or 1/4 are always white preventing the achievement of all white or all black uppermost. It works on a very simple principle that can be the basis of a game. One can also work out any of the rotations on paper if you remember that whenever a row or column is rotated, all the white colors in the row or column are replaced by black and vice versa. With this in mind can the reader work out a simple configuration that can be generalized and will always be a maximum asymmetry? Maximum asymmetry is where at least 1/4 of the cubes are always a different color. It becomes a little more complicated to start with a random array and determine what percentage of the array is asymmetric.

When 90° rotations are allowed any orientation (of the six allowable orientations) of any cube in any row and column can be achieved in combination with any orientation of any of the other cubes in the matrix. In other words the 90° allowance gives



· FIGURE 1

the array complete mobility. However this is easier said than done. It is quite difficult to get the array from one given combination to another by 90° rotations if it is a 3×3 or larger matrix. An exact formula for figuring out what the minimum number of moves would be for getting a given array from one combination to another has not been worked out. It is fairly certain that one exists or is possible. Perhaps the following result will give some idea as to how to attack this problem.

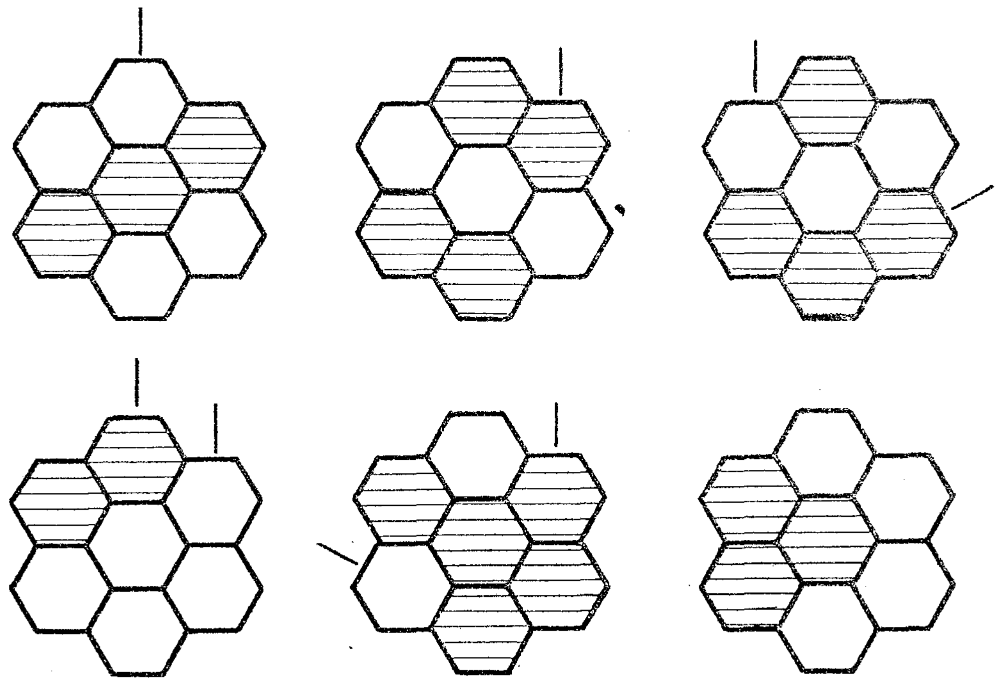


FIGURE 3

Instead of using cubes, we could have used hexagonal plates—if we can be satisfied with 180° rotations Figure 3 shows a set of rotations performed on an array of 7 hexagons. Rotations can be made along three directions in hexagonal arrays, but the analogy with three dimensions does not go much further. From the series of rotations in Figure 3, can the reader determine if in any given hexagonal array any combination of black and white hexagons is possible by rotations along the three allowed directions?

A simple game called Arrow can be played using a 2×2 or 3×3 array. Mark arrows on the cubes as shown in Figure 4. Play starts with all cubes placed so the arrows are oriented identically on each cube. Rotations are restricted to 90° in either direction. Each player may rotate either rows or columns in his turn. There are two players and they attempt one of two combinations of the arrows. If one player tries to get two arrows to point toward each other, the other player tries to get two arrows to point away from each other. A set of moves in a 2×2 game which results in a win for arrows pointing toward each other is shown in Figure 4.

Can the game always end in a win for the player to move first?

The investigation of Cubo-Caibo ends here. The only additional thought concerns using more complex solids in a rotational matrix. One which comes immediately to mind is the rhombic dodecahedron. It has two plane sections which could be used for an array. One intersects 4 faces in a square and the other intersects six faces in a

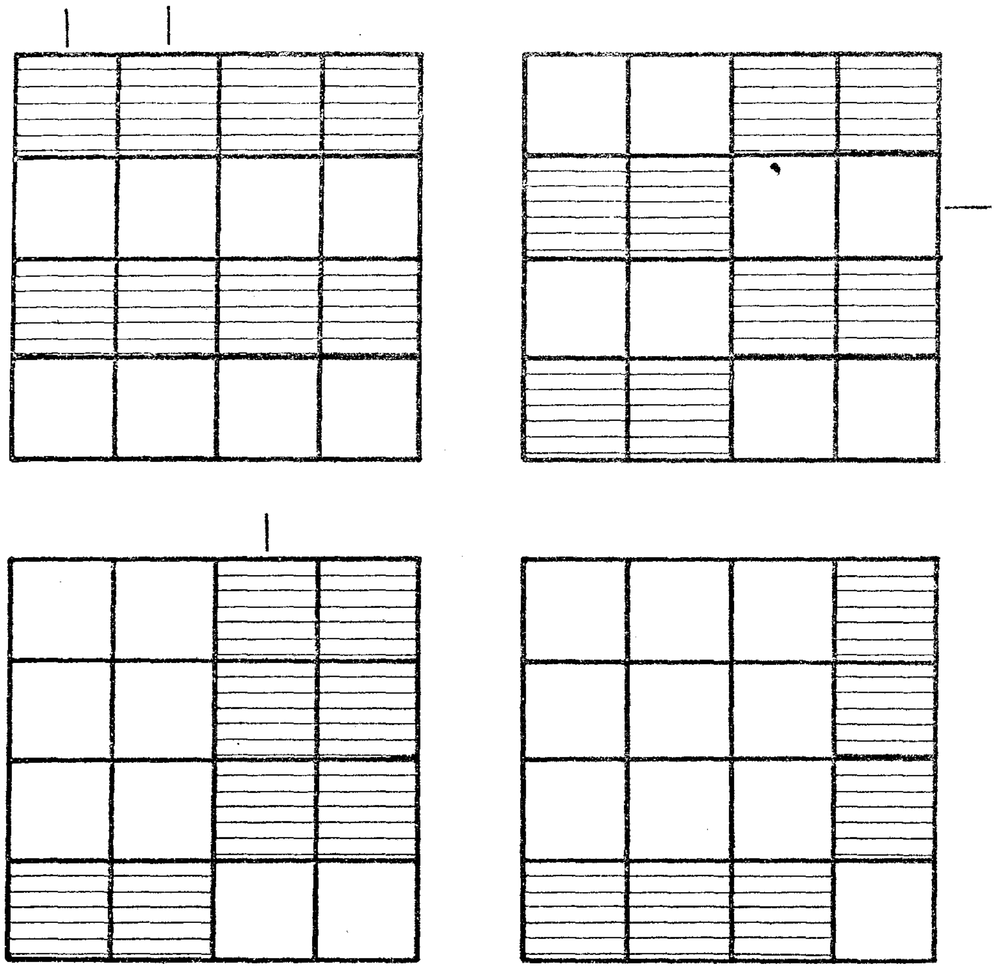
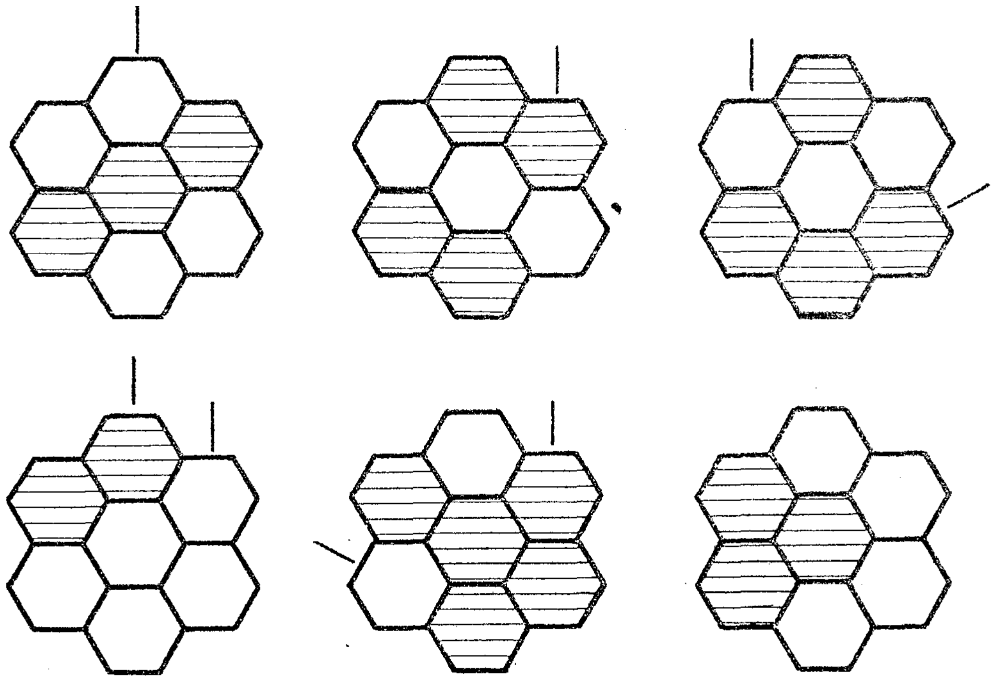


FIGURE 2

If rows and columns are numbered, rotations can be kept track of. When a set of rotations are repeated in exactly the same way, enough times, the matrix always returns to its starting position. If the rotations in the set or cycle are all in the same direction (clockwise for instance) along rows and columns the matrix repeats after completing 3 cycles of the set of rotations. If some of the rotations are in the opposite direction, the matrix repeats after 12 cycles of the set of rotations.

It is interesting to note that in a small array, say 4 × 4, there are many trillions of combinations of the uppermost faces. In fact, there are 6^{16}, or 2,821,109,907,456, in the 4 × 4 array if there are six different colors on the cubes of the array. If a machine could be made to recognize color combinations, the matrix could be used as a combination lock. First set it with all the cubes oriented identically, then make the right combination of rotations along rows and columns to make the machine open the lock.



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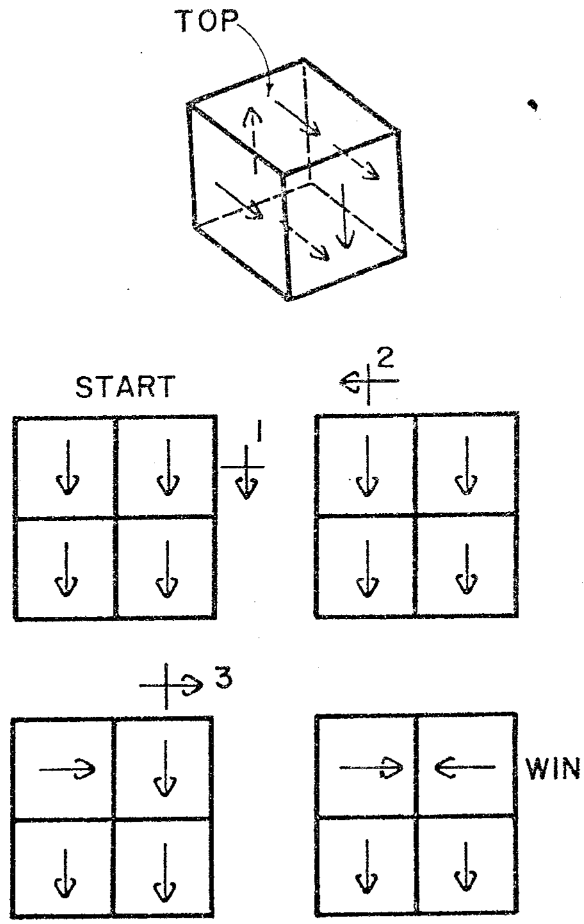

FIGURE 4

hexagon. The plane section which intersects 4 faces would not be interesting since it would be restricted to 180° rotations. It is therefore equivalent to a Cubo-Caibo matrix where rotations are restricted to 180°.

The other section forms a hexagonal matrix. It allows either a 180° rotation or a rotation of 72°, and any rotation of $n(180^\circ) + 72^\circ$. Such a matrix will have a good deal more complexity than the cubic one. Rhombo-Rhymbo must end here, however, and await its turn to future rotations.