Reprinted with Permission from Recreational Mathematics Magazine, No. 11, Oct. 1962, pp, 3-5, Box 35, Kent Ohio, Editor Joseph S. Madachy, a prolific creator of Rec. Math., such as the magazine JRM and many puzzles. He was a great enthusiast. *(This was the first publication of my research efforts into he topology of digital or quantized twist puzzles. I am still doing these experiments Jan, 2019. Martin Gardner liked the Hexaflexatetrahedron and said it emulated hexaflexagons.)*

FLEXAHEDRONS*

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(A student at Fort Hays Kansas State College.)

If anyone has three hands and a little patience he can manufacture the structures I will describe here which I discovered in the Fall of 1961. I was playing with the possibilities of flexagon, or Jacob's Ladder, hinges. After much fumbling and bumbling I managed to get six tetrahedrons connected as shown by the process of weaving in Figure 3. The finished product looked like Figure 7. I used thin aluminum cut into strips to make the tetrahedrons weaving along the dotted line shown in Figure 1 and Figure 2. String was first used, but I found that this was the hard way of going about it. Paper bands worked better, giving much less trouble in connecting the tetrahedrons. Cardboard will work as well as aluminum to make the solid, but aluminum is more durable. I call this chain, unmercifully, a hexaflexatetrahedron. All of its movements seem to be regular and symmetrical. Besides the flexing from a rectangular shape to a hexagonal shape, shown in Figures 4, 5, 6, and 7, it can be got into other shapes where it does not flex but stops short and the only way out is to go "backwards."

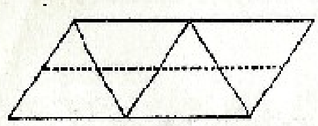

Figure 1

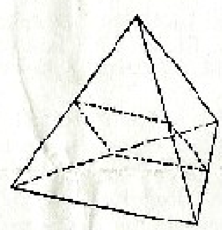

Figure 2

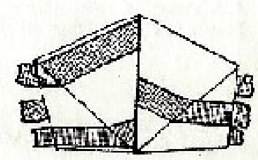

Figure 3

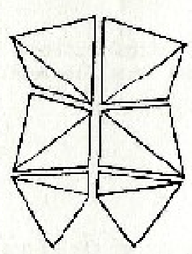

Figure 4

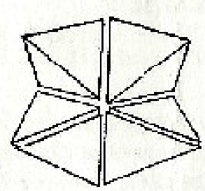

Figure 5

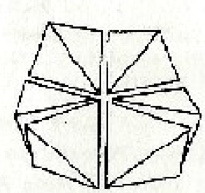

Figure 6

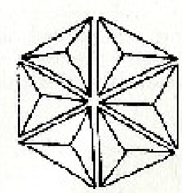

Figure 7

I also connected four, seven, and eight tetrahedrons into chains. The four tetrahedrons did nothing but roll around each other. Since there was no possibility of a space between them they could not possibly have had more than one definite movement. The eight tetrahedrons were irregular in movement, though quite fascinating. The seven-tetrahedron chain was formed and then given a half-twist before connecting the ends. Its movement was very limited and compared to the others it resembled an old Model T Ford beside a new Edsel. I suppose had I connected enough of an odd number of them by the above method they would have flexed but the only advantage I could see was that the combination of faces would constantly change.

Next I connected four cubes by following the dotted lines shown in Figures 8 and 9. The right triangles cut off by the two dotted lines in Figure 8 are not needed in the structure as they do not enter into the active movement. I then connected six of them and, lo and behold! I had a whole new fascinating set of movements to play with. In Figure 10 the six cubes are in the position where you fold them over and over to flex them until it becomes quite boring and you want something different.

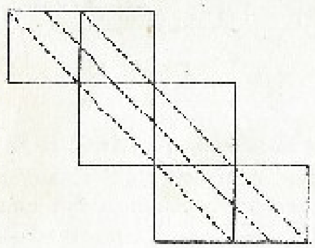

FIGURE 8

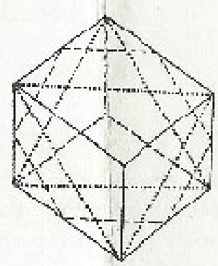

FIGURE 9

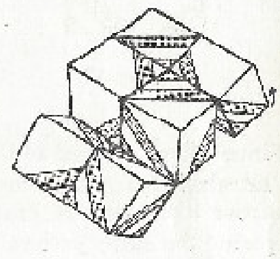

FIGURE 10

Figures 11 to 15 show how to get the apparatus into a position where it has a wholly new way of flexing. By flexing I mean that each object is made to roll around and the bands to move over them. There is a similarity in the symmetry and flexing property here resembling the tetrahedron chain of six. There are at least two locked positions it can achieve and this is also similar to the tetrahedron chain of six. I observed none of this in the octahedron chains which were constructed. It should be noted that only in the tetrahedron and the hexahedron can the bands be made to pass over all the fences on the solid. The other three Platonic solids can be connected in similar fashion, but no such fascinating flexing properties are observed—they only roll around and around.

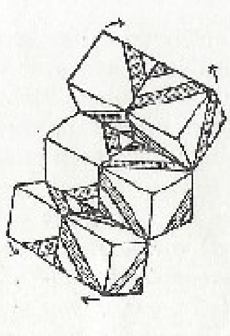

FIGURE 11

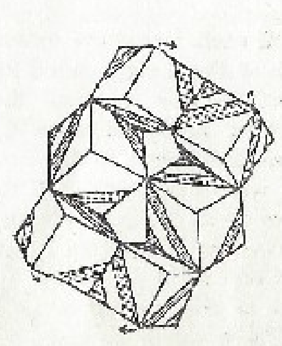

FIGURE 12

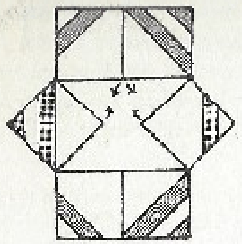

FIGURE 13

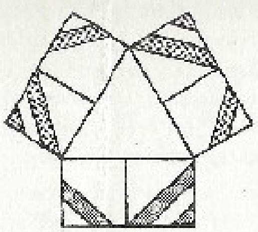

FIGURE 14

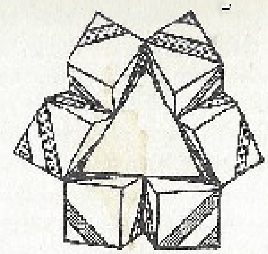

FIGURE 15

Since I have arrived at these flexing solids from a study of flat flexagons I tried to flatten the tetrahedrons and connect them. By using a strip of four isosceles right triangles to construct it, instead of four equilateral triangles, the tetrahedron becomes a flat square. Following the same pattern of connection as in the regular tetrahedrons, I connected six of these flattened tetrahedrons, or squares.

The only similarity I could discern between the squares and the tetrahedrons was the rolling around—otherwise they differed considerably. Getting nowhere with flattened-out tetrahedrons forced me onto another track. I decided that since the triangles in the flattened-out tetrahedrons were the same as the triangles used in the "strip part" of the cube and since the cube has two more faces than the tetrahedron I should flatten a cube out to a hexagon and use the resulting 120° isosceles triangles. Figure 16 depicts this strip. Figure 17 shows the completed figure.

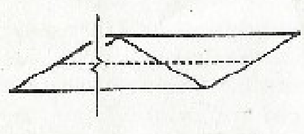

FIGURE 16

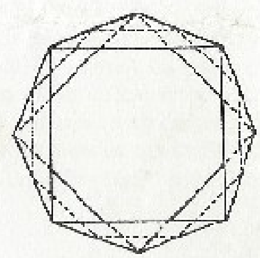

FIGURE 17

If the plane of each face were extended a kind of octohedron-like solid would result. I connected four of these eight-sided figures and found that it had a new way of flexing but only one definite way as in all the other connections of four. There is one locked position obtained by giving opposite figures in the chain a twist in opposite directions. When this is done they come to the closest possible contact with each other.

My next task was to connect six of these eight-sided figures. It took monstrous patience but I managed to do it. I advise anyone trying to do so for himself to have help when it comes to the process of connecting it. However, it is well worth the effort—the fascinating movements are never to be forgotten. I call it a hexaflexaoctahedron. While the chain was intact I was able to observe numerous locked positions. In fact it was locking more than it was flexing. There was a distinct similarity between it and the cubes and tetrahedrons but because of the size of the angles involved and the resulting forces, it had a tendency to come apart and I soon found myself amidst a pile of eight-sided figures whose only remaining property was to exasperate!

What must follow is some mathematical formulation of the properties of these flexing solids so that the properties of each succeeding one can be predicted. The next type solid would give ten sides and 144° in each triangle composing the strip since flattening out the eight-sided figure produces the 144° in the interior angles of the resulting octagon. I do not know the truth of this since I have not been able to reduce any of it to mathematics—my brainchildren have surpassed my methods. Perhaps the future will produce results.