THE SET OF MAGNECURVES AND THEIR PROGENEY,
IN UNIMODE, MULTIMODE, ETC.

ROADCURVES

(a special look at cycle double covers)

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Some time ago I submitted a problem to a math magazine. Dr. Richard Nowakowski editited it, and thought highly enough of it to publish it(American Mathematical Monthly, Jun-Jul,2000, p563). Since it was just a problem for solution it barely touched the surface of what can be done with the idea. I could not find a reference showing it had been solved. Keltahedra present almost the same identical problem(see puzzleatomic.com). The devices used to elucidate the problem were dubbed Magnecurves. The idea is that you draw two directed paths along each edge of a graph with the directions always going opposite ways. This seemed kind of like the way two bar magnets would align with opposite poles always together. Figure 1 below shows this idea executed on the edges of a cube, which has been drawn in perspective. We ignore the three dimensional quality of the cube and just draw the lines using a node or vertex rule. In this case the rule is make a turn when you arrive at a vertex, and continue along the new edge without crossing over any edges. Continue in this fashion until you complete a circuit. We could consider this path to be an element of a subset of the Magnecurves and call it a Roadcurve since the paths look similar to the lanes of a two lane road network.

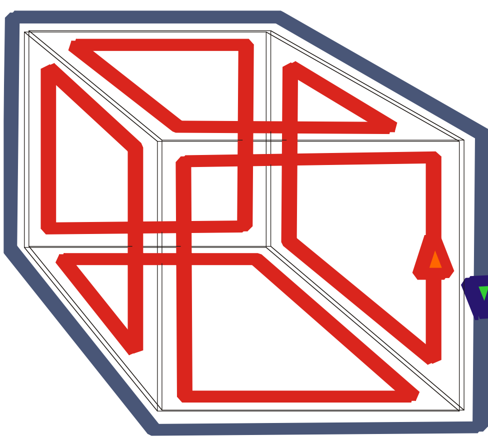

Figure 1

Road Curves

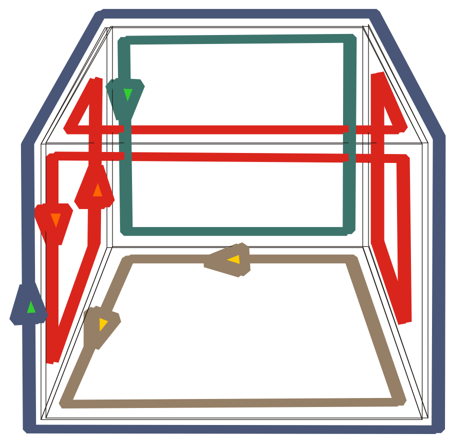

Figure 2

Note that with two different perspective drawings of a cube we get two different sets of Roadcurves. In Figure 1 we ended up with two Roadcurves and in Figure 2 we ended up with four Roadcurves.

The question is, why do we have two different sets of routes when both drawings are simply a perspective of a cube, and we followed exactly the same road rule in both cases? This puzzle is left to the reader to pore over in a pleasant moment of recreational mathematics. Hint: think about picking up each cube out of the two dimensional realm and into the three dimensional realm as an actual solid cube. It brings up the idea that looking at a flat drawing causes the Magnecurve Uncertainty Principle, MUP. Indeed, this might be one of the reasons great art is great. Great art displays, in our subconscious, the more intricate, and interesting road trips. Of course, the fact is that this is dead serious, and from now on cannot be ignored by any good, practising, gestalt artist.

MAGNECURVES AND THEIR PROGENEY, CONTINUED

ROAD CURVES

Figure 3 shows the three Roadcurves for a 2x2x2 cube graph with the same kind of perspective as Figure 1. With the 1x1x1 cube graph in Figure 1 there are two Roadcurves while this 2x2x2 shows three. Is there some way to generalize this so that we can say that for any nxnxn cubic graph in a given perspective there are f(n) curves? We can be more general and ask how many RC exist for any pxqxr cubic graph in perspective P? A nice puzzle to while away at in some relaxed moment. Such questions can also be asked of any kind of periodic space lattice work of connected edges and vertices.

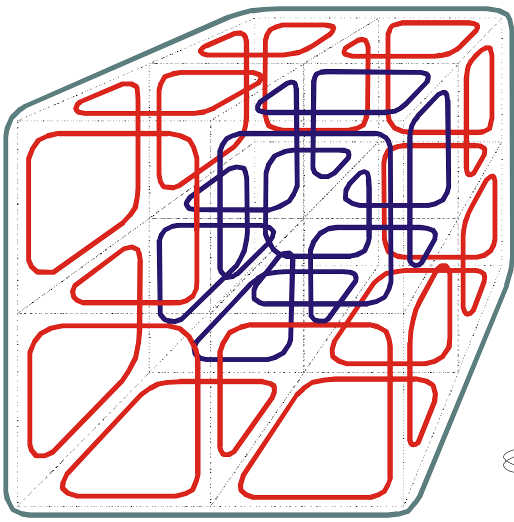

Figure 3

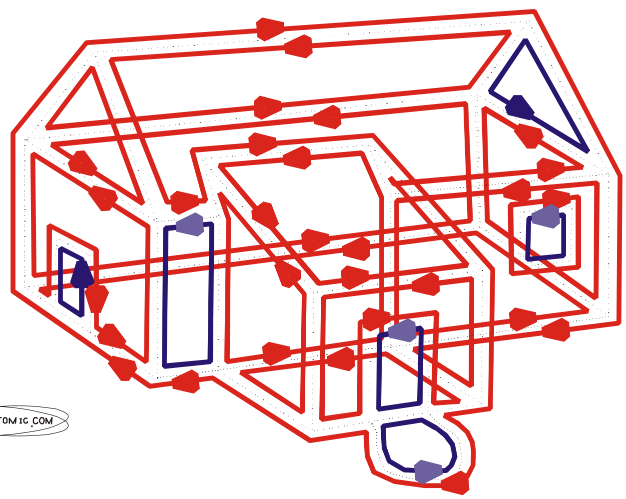

Figure 4

An artist might find the Roadcurves useful to apply to a drawing or a painting, hiding his method with various subtle techniques, thereby allowing the viewer the pleasant, or perhaps unsettling experience, of trying to figure out the regularity competing with the art. Figure 4 illustrates a start on this technique. Make a simple graph of a building and then produce its Roadcurves. After that you could erase the graph and paint between the Roadcurves, thus showing that the edges of the thick painted 'roads' actually form road curves for any perspective graph with transparent faces, but this is might be the first time that we realize this fact!

Figure 4 also shows how the Roadcurves are faithfully executed with the cars always staying to the left, in true British fashion. We have only to select a driving rule to produce a consistent, and perfect set of mathematical, or perhaps a more appropriate word is topological, road loops. We can be assured that there will always be exactly two oppositely directed paths traversing each and every edge of the graph.

MAGNECURVES AND THEIR PROGENEY, CONTINUED

CURLCURVES

What will happen to the Roadcurves if we change the rule at some or all the vertices? For instance, our intersection crossing rule for the Roadcurves is 0, curl rule=0, or cross no intersecting edges when coming to an intersection. Just go down the next road available in your lane. Suppose be have curl rule=1, or 2, etc. If 1 then we mean to cross over exactly one intersecting edge before continuing in our lane. You might think that this new rule would just produce the same curves. This is not so, as we can see in the examples in Figure 5, where the rule is 0, 1, and 2 for the three identical graphs.

These new curves are kind of like Roadcurves but to distinguish them we will call them Curlcurves. Curlcurves are a subset of the Magnecurves. They become especially interesting if the curl rule can be different at different intersections. But when the rule is the same for all intersections, we can predict how many of the different curl rule networks, applied to a given graph can produce a different overall network of paths. The overall network ignores the curling and just considers how the path meanders from vertex to vertex. It turns out that the number of different overall Curlcurves will be equal to the product of the mutually prime degrees of the vertices. Applying this to the graph in Figure 5 we have vertices of degree 2, 3, and 4. Since 3 and 4 are mutually prime, the number of different Curlcurve path sets will be 12, and then start repeating in multiples of 12. Curl rules=0,1,2,3, …11 can each produce different path sets, and then curl rules 12,13,14,…23 will produce the same overall pattern as 0,1,2,...11, and so on, ad infinitum. Can you see why it works this way?

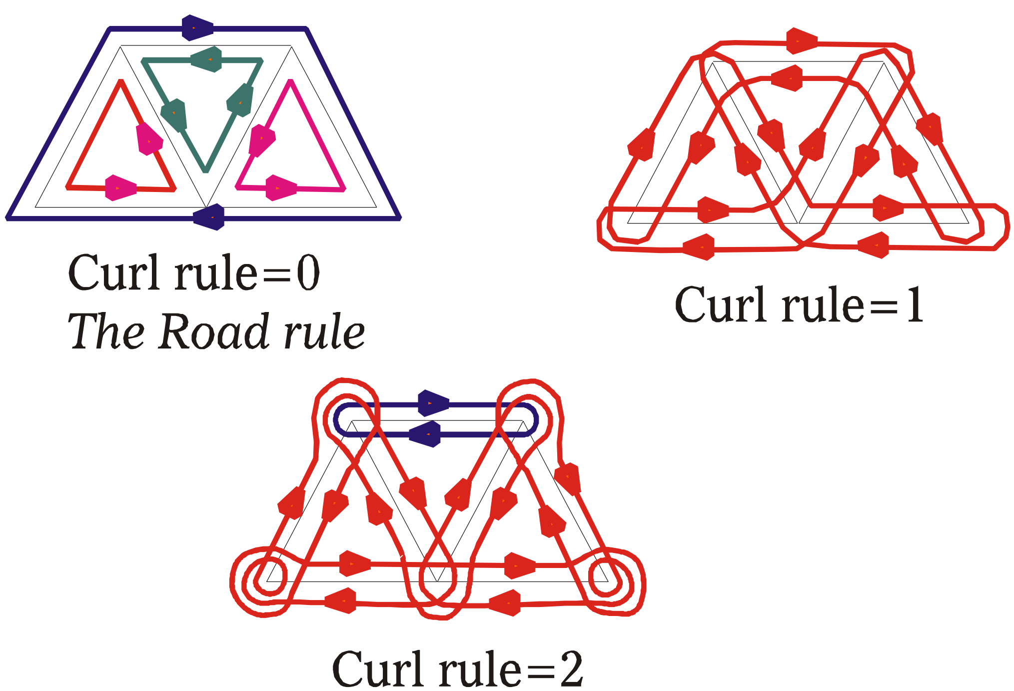

Figure 5

MAGNECURVES AND THEIR PROGENEY, CONTINUED

KNOTCURVES

So far the subsets of Magnecurves have focused upon 2D curves and 2D perspective graphs of 3D graphs. Can we devise rules that always result in oppositely directed paths on a 3D graph? We can't use a 2D perspective edge crossing rule because when we do this the paths produced are variable as shown in Figures 1 and 2. For a true 3D rule that always produces the same path set, we could color the edges different colors. Then the rule says move along the colored edges, A,B,C...A,B,C.. until a path is closed. If the vertex is a dead end of degree one, make a U turn and go back along it. If a color does not exist at a vertex then take the next color specified by the rule. Figure 6 shows a 3D color rule applied to a cube graph. We get four paths, each of length 6, and all four similarly shaped. Numbered edges would work the same way. For the regular polyhedrons can you always find a way to color the edges to produce similar shaped paths? How about the regular polytopes, etc.?

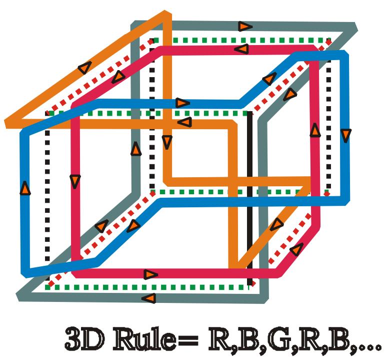

Figure 6

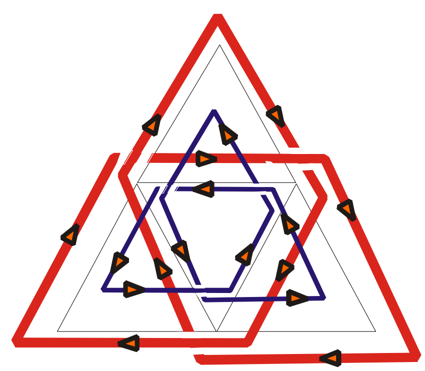

Figure 7

Define Knotcurves on a two-dimensional graph by inventing rules about how a path crosses over and under another path, wherever a crossing occurs. Figure 7 shows a trefoil Knotcurve. By making paths into ribbons we could twist the ribbons an even number of times and create more complex knots and surfaces like Seifert surfaces.

The Magnecurve set is extendable. Just devise rules that provide an input and an output from each edge to each vertex in the graph. The path leaving is the output and the incoming path is the input. This said we could make some analogies with the nerve circuits of our minds. A neuron is a vertex with outputs and inputs along edges. A curl rule might be like a time delay combined with an 'xor' on an edge. A color rule would be like a chemical 'and' gate along an edge. When we make the paths wind around the edges an even number of times as they traverse the graph, we call this Magnecurve subset Vinecurves. Making them into ribbons, and twisting them an even number of times, sort of does the same thing, except that the vines are actual curves whereas the ribbons are surfaces and only their edges are Vinecurves. Thus, we see that the subsets of Magnecurves can be combined to produce multimode curves. For instance, Curlcurves could be combined with Knotcurves and Vinecurves to give CKVcurves. If a curve is not a combination of two or more kinds it is a unimodal Magnecurve. With simple curve classifications we can create a little universe of curves to walk along and while away our pleasant Recreational moments of Mathematics.

MAGNECURVES AND THEIR PROGENEY, CONTINUED

MORE ABOUT 3D RULES

You ask, how can we simulate the curl rules on a 3D graph? Simple, just skip, according to the rule, the edge colors or edge numbers of the edge sequence occurring the node. For instance, if there are a total of four edge colors then the sequence at a degree 4 node will be R,G,B,P,R,… and at a degree 3 node it will have only a three color sequence such as R,G,B,R,… ,… and at a degree 2 node it will have only a two color sequence such as R,G,R,… Thus if the Skip rule is 2, and you are traveling along an R edge to a 4 node the next color edge chosen would be P. If you were traveling along R to a 3 node the next color edge chosen would be R(make a U turn and go back the way you came). If you were traveling along R to a 2 node the next color edge chosen would be G. This gives the same number of, possibly, different overall networks as stated above for curl rules, namely the mutually prime product of all the nodes. We say 'possibly' because a proof of uniqueness will not be covered in this short paper.

BEYOND MAGNECURVES

It is very easy to create large, complex yet highly organized graphs by thinking of any graph as either a factor or a product. For instance, you can take a simple n by m orthogonal graph and repeat it in a 2 dimensional (mn)(mn) array, where n and m should be mutually prime for the best results. Take another copy of this repeated graph rotate 90 degrees and place over the original to make a product graph. Now use the product graph to make Magnecurves. Using these techniques it is possible to produce amazingly beautiful graphics on a computer screen. The products can be varied. You could rotate the factor graph or flip it, etc. as you create the columns or rows of the semi product graph, (mn). You could flip over the overlapping graph before overlapping. In three dimensions even more ways of flipping and rotating are possible. Any number of factor graphs can be multiplied together to produce product graphs in two or three dimensions. You can give additional properties to the paths. For instance you could assign a path a starting position point and value. This circuit with a start node could be likened to a memory. It could cause other paths to come to life with their own starting point and value, like a mental cell firing to cause other cell firings in the network. Magnecurves are just a subset of these freewheeling new Rulecurves which need not obey the two directions rule. Rules can be applied to both edges and nodes. A curve could be open ended, or go into an infinite loop at one end, or loop for a while then meander somewhere else, and pass over the same edges and nodes multiple times Rules can be made to change depending on previous rules encountered by the path. The full study and science of these interacting paths and networks is called Pathematics.

Magnecurves are able to produce artistic and beautiful patterns in many ways. In league with properly created product graphs Magnecurves, and their offshoots, can produce large complex and self organized networks with well defined paths or circuits. Because we need only start with simple factor patterns and a small rule base a small compact computer program will generate great complexity at will. Using the concept of overlapping factor products(they could be number matrix's, matrix's of formulas, along edges and nodes etc.) you could solve difficult problems not unlike the concept of superposition used to design quantum computers. In fact such matrix systems are already in use. Chips have been made with arrays of sensors and chemicals are spread over the array in a product pattern. The sensors with the best values give the 'answer chemical' sought for. This is similar to the idea of superposition. One could think of a large array of computer chips, say 1000x1000. Problems are fed to the array according to a product array. The answer is spit out immediately from the 'cells' where the product pattern satisfies the needed inputs or superposition provided by the product array. But mathematicians have been using something quite similar to superposition for centuries. Perhaps it sort illustrates why quantum superposition is so powerful. There are hundreds of examples. For instance Cantor's diagonal proof, or the superposition that occurs when you align two infinite sequences to cause everything but the sought for answer to cancel out!!!

Notes: (5-2020) This short paper was submitted and published by the Gathering for Gardner 2012. Since this was published I have become aware that these Magnecurves might also be referred to in mathematics as a double cover for a graph. The Magnecurve designation came about to always try to find a system where all parallel paths point in opposite directions, like a magnetic kind of polarity. It was also a goal to find a Magnecurve that could covet the entire graph with one circuit. Doug Engel

Refs: Double cover link There are also many excellent puzzles and math investigations of double covers.